

# Dilaton Tadpoles, Warped Geometries and Large Extra Dimensions for Non-Supersymmetric Strings

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## Abstract

We analyze the backreaction of dilaton tadpoles on the geometry of non-supersymmetric strings. After finding explicit warped solutions for a T-dual version of the Sugimoto model, we examine the possibility of realizing large extra dimension scenarios within the context of non-supersymmetric string models. Our analysis reveals an appealing mechanism to dynamically reduce the number of flat, non-compact directions in non-supersymmetric string theories.

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## 1. Introduction

Non-supersymmetric string compactifications [1-10] have recently attracted attention in particular due to the possibility that in open string models the string scale is not necessarily closely tied to the Planck scale [11]. In models containing lower dimensional D-branes, extra large transversal directions can give rise to a large Planck scale while leaving the string scale essentially a free parameter. In these large extra dimension scenarios with a string scale in the TeV range, supersymmetry is not necessarily needed for protecting the gauge hierarchy. Thus, TeV strings provide the natural arena for non-supersymmetric string models. The last year has seen intense efforts in non-supersymmetric string model building. Of particular interest are models with brane supersymmetry breaking [5,6], where the tree level bulk still preserves some supersymmetry while supersymmetry is broken on the D-branes. Phenomenological questions like the embedding of the standard model have been addressed in [12], while stability issues were intensively discussed in [13].

All these models have in common that they feature a non-vanishing dilaton tadpole, meaning that the string equations of motion are not satisfied by the factorized metric  $M_4 \times K_6$  and a constant dilaton. In particular, for the open string models the dilaton tadpoles appear at disc level and their backreaction should be taken into account to come closer to the true quantum vacuum of the theory. In the deformed background the tadpole has disappeared [14]. To find the true perturbative quantum vacuum, one would need to solve the string equations of motion to all string loop levels. However, for non-supersymmetric strings this is far beyond the reach of computational power.

Extending the work of [15], we will solve the effective equations of motion for the string background in a prototype example. In particular, we are interested in a simple enough toy model with D-branes allowing transversal directions. Such a situation is provided by a T-dual version of the Sugimoto model [5], where positive tensions are localized on two fixed points on a circle. Taking the backreaction of the dilaton tadpole into account, we obtain a new class of non-trivial warped geometries and dilaton profiles. In these backgrounds the expressions for the lower dimensional Planck scale and gauge couplings in terms of the string scale and the internal geometry are drastically changed. Therefore, it is a legitimate question whether these modified relations still allow to disentangle the string scale from the Planck scale by increasing some radii in the internal space.

Intriguingly, the solutions we find only admit a finite lower dimensional effective theory with zero cosmological constant, if the dimension of the flat, non-compact space is not

bigger than the critical value six. Thus, in non-supersymmetric string theories quantum corrections can reduce the number of flat space-time dimensions, leading to an appealing mechanism to explain why we live in four dimensions.

This paper is organized as follows. In section 2 we review the construction of the Sugimoto model and introduce the T-dual model we are going to consider in the following. In section 3 we present our solutions for the backreaction of the dilaton tadpole on the geometry and the dilaton profile. In section 4 we compute effective lower dimensional gravity and gauge couplings and discuss the issue of large extra dimensions in this backgrounds. In section 4 we end with some conclusions.

## 2. Sugimoto Model

The prototype example of an open string model featuring brane supersymmetry breaking is the so-called Sugimoto model [5]. It is a non-supersymmetric version of the Type I string. Whereas the supersymmetric Type I string contains orientifold planes of negative tension and RR charge, the Sugimoto model contains orientifold planes of positive tension and RR charge. This modification does not change the Klein bottle amplitude at all, implying that at closed string tree level the bulk still preserves supersymmetry. However, in order to cancel the dangerous RR tadpole one has to introduce 32 anti-D9-branes, which of course also have positive tension. Thus, even though the RR charge is cancelled, the background contains positive tension branes generating a non-vanishing dilaton tadpole. Moreover, the Möbius amplitude is non-vanishing, so that the model explicitly breaks supersymmetry. Note that there does not exist any way of cancelling the RR tadpole by a supersymmetric configuration of D-branes.

Computing the ten-dimensional spectrum of this model, one finds no tachyon and massless vectors of the gauge group  $USp(32)$  in addition to a massless fermion in the antisymmetric representation. As expected from RR tadpole cancellation, the anomalies cancel.

The Sugimoto model already features one of the notorious problems of non-supersymmetric strings, namely the presence of a dilaton tadpole, respectively a non-vanishing cosmological constant. Thus, beyond the leading order in the string coupling, a flat ten-dimensional Minkowski space and a constant dilaton are not solutions of the string equations of motion. In order to come closer to the true quantum vacuum, in the next to leading order one should take the backreaction of the dilaton tadpole into account.

For the ten-dimensional Sugimoto model with space-time filling  $\overline{D9}$ -branes this has been done in [15], where it was shown that the effective equations of motion admit a solution with less Poincaré symmetry and a non-trivial dilaton profile. More concretely, the solution found there was a warped metric with nine dimensional Poincaré symmetry featuring spontaneous compactification of the tenth direction and localization of gravity to nine dimensions.

In this paper we are interested in examining whether taking this backreaction into account, it is still possible to disentangle the Planck scale from the string scale. This is a non-trivial issue as the leading order relations

$$M_{Pl}^2 \sim \frac{M_s^8 V_d V_{6-d}}{g_s^2}, \quad \frac{1}{g_{YM}^2} \sim \frac{M_s^d V_d}{g_s} \quad (2.1)$$

for the four dimensional Planck mass and the gauge couplings get modified. In (2.1)  $V_d$  denotes the volume longitudinal to the D-branes and  $V_{6-d}$  the volume transversal to the D-branes.

For the warped metric found in [15] the four-dimensional Planck scale and gauge coupling take the values

$$M_{Pl}^2 \sim M_s^{\frac{17}{2}} V_5 R_c^{\frac{3}{2}}, \quad \frac{1}{g_4^2} \sim M_s^{\frac{11}{2}} V_5 R_c^{\frac{1}{2}}, \quad (2.2)$$

with  $R_c$  denoting the effective size of the spontaneously compactified direction  $x_9$ . With gauge coupling of order one, the relations (2.2) imply  $M_{Pl}^2 \sim M_s^3 R_c$ , so that even with space-time filling  $\overline{D9}$ -branes a large extra dimension scenario is possible (at least in the next-to-leading order approximation).

In the following we will continue to investigate these quantum corrected space-times by studying a T-dual version of the Sugimoto model. After performing one T-duality along the tenth direction, denoted  $y$ , one gets a model with two positively charged O8-planes located at the two fixed points of the reflection  $y \rightarrow -y$ . We are cancelling the RR charge locally by putting 16 anti-D8 branes on each fixed point, chosen to be at  $y = L/2, 3L/2$ . In one loop,  $e^{0\Phi}$ , order, this appears to be a stable configuration. Since the Klein-bottle and the annulus amplitude vanish, the leading order force between two O8 planes, respectively two  $\overline{D8}$ -branes vanish. Only the Möbius amplitude is non-vanishing and leads to an attractive force between an O8-plane and a  $\overline{D8}$ -brane.

Note that the resulting model is nothing else than a non-supersymmetric version of the Type I' string. As in the original Sugimoto model, we are left with non-zero dilaton

tadpoles due to the positive tension localized at the two fixed points. In string frame the effective action for the metric and the dilaton is

$$S_S = \frac{M_S^8}{2} \int d^{10}x \sqrt{-G} e^{-2\Phi} [R + 4(\partial\Phi)^2] - 32T \int d^{10}x \sqrt{-g} e^{-\Phi} [\delta(y - \frac{L}{2}) + \delta(y - \frac{3L}{2})], \quad (2.3)$$

where  $g_{ab} = \delta_a^M \delta_b^N G_{MN}$  denotes the 9-dimensional metric induced on the branes. We take  $M, N$  to run over all spacetime and  $a, b$  over the longitudinal coordinates. Note that the brane tension  $32T$  is the same on both fixed points and that due to the cancelled RR-Flux we can set the RR nine-form to zero.

### 3. Solutions

In this section we will construct solutions of the equations of motion resulting from the action (2.3). Note that these equations are very similar to those encountered in the dilatonic Randall-Sundrum scenario [16]. The essential difference is that we have two branes with positive tension on a compact space.

To study a more general class of solutions we first compactify the string theory on a  $(8 - D)$  dimensional torus of volume  $V_{8-D}$ . Thus, in string frame we split the metric as

$$ds_{10,S}^2 = ds_{D+2,S}^2 + \sum_{m,n=D+3}^{10} \delta_{mn} dx^m dx^n. \quad (3.1)$$

We transform the resulting effective action via  $G_E = e^{-\frac{4}{D}\Phi} G_S$  into Einstein frame to obtain

$$S_E = \frac{M_S^8 V_{8-D}}{2} \int d^{D+2}x \sqrt{-G} [R - \frac{4}{D}(\partial\Phi)^2] - 32T V_{8-D} \int d^{D+2}x \sqrt{-g} e^{\frac{D+2}{D}\Phi} [\delta(y - \frac{L}{2}) + \delta(y - \frac{3L}{2})]. \quad (3.2)$$

The resulting equations of motion for the dilaton and the metric are

$$\begin{aligned} \partial_M(\sqrt{-G} G^{MN} \partial_N \Phi) &= \frac{D+2}{4} \lambda \sqrt{-g} e^{\frac{D+2}{D}\Phi} [\delta(y - \frac{L}{2}) + \delta(y - \frac{3L}{2})] \\ R_{MN} - \frac{1}{2} G_{MN} R &= \frac{4}{D} \left( \frac{1}{2} G_{MN} G^{PQ} \partial_P \Phi \partial_Q \Phi - \partial_M \Phi \partial_N \Phi \right) + \\ &\quad \lambda g_{ab} \delta_M^a \delta_N^b \sqrt{\frac{g}{G}} e^{\frac{D+2}{D}\Phi} [\delta(y - \frac{L}{2}) + \delta(y - \frac{3L}{2})], \end{aligned} \quad (3.3)$$

where we have introduced  $\lambda = 32T/M_s^8$ .

Due to the fact that both tensions at  $y = L/2$  and  $y = 3L/2$  have the same sign, there does not exist a solution to these equations with  $(D+1)$  dimensional Poincaré invariance. This is in agreement with the sum rules recently derived in [17]. Therefore, the best we can try is to look for solutions with  $D$ -dimensional Poincaré invariance, for which we make the following warped ansatz

$$ds_{D+2}^2 = e^{2M(r,y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2N(r,y)} (dy)^2 + e^{2P(r,y)} (dr)^2. \quad (3.4)$$

We also assume that  $\Phi$  depends only on  $r$  and  $y$ . Note that this ansatz assumes that the cosmological constant in the effective  $D$ -dimensional theory vanishes. As usual, one first constructs solutions of eqs.(3.3) in the bulk and then imposes the jump conditions due to the branes at  $y = L/2, 3L/2$ . In order to satisfy these jump conditions we are led to choose warp factors that depend separately on  $r$  and  $y$ . More precisely, we take

$$M(y, r) = A(y) + X(r) \quad , \quad N(y, r) = B(y) + Y(r) \quad , \quad P(y, r) = C(y) + Z(r). \quad (3.5)$$

Likewise, we choose  $\Phi$  to depend separately on  $r$  and  $y$  and write

$$\Phi(y, r) = \varphi(y) + \chi(r). \quad (3.6)$$

It is straightforward to insert the above ansatz into (3.3) to find the equations of motion. Simplifying the notation by introducing

$$\Delta = \left[ \delta(y - \frac{L}{2}) + \delta(y - \frac{3L}{2}) \right], \quad (3.7)$$

the dilaton equation of motion reads

$$\begin{aligned} & \left[ \varphi'' + (DA' - B' + C')\varphi' \right] + \\ & e^{2(B-C)} e^{2(Y-Z)} \left[ \ddot{\chi} + (D\dot{X} - \dot{Z} + \dot{Y})\dot{\chi} \right] = \frac{D+2}{4} \lambda \exp \left( B + Y + \frac{D+2}{D} \Phi \right) \Delta, \end{aligned} \quad (3.8)$$

where primes and dots refer to derivatives with respect to  $y$  and  $r$  respectively. From the  $\mu\nu$  component of the Einstein equations we obtain

$$\begin{aligned} & e^{2(B-C)} e^{2(Y-Z)} \left[ (D-1)\ddot{X} + \frac{D(D-1)}{2}(\dot{X})^2 - (D-1)\dot{X}\dot{Z} + (D-1)\dot{X}\dot{Y} + \right. \\ & \left. \ddot{Y} + (\dot{Y})^2 - \dot{Y}\dot{Z} + \frac{2}{D}(\dot{\chi})^2 \right] + \\ & \left[ (D-1)A'' + \frac{D(D-1)}{2}(A')^2 - (D-1)A'B' + (D-1)A'C' + C'' + (C')^2 - \right. \\ & \left. B'C' + \frac{2}{D}(\varphi')^2 \right] = -\lambda \exp \left( B + Y + \frac{D+2}{D} \Phi \right) \Delta. \end{aligned} \quad (3.9)$$

The  $rr$  component gives

$$\left[ DA'' + \frac{(D+1)D}{2}(A')^2 - DA'B' + \frac{2}{D}(\varphi')^2 \right] + e^{2(B-C)} e^{2(Y-Z)} \left[ \frac{D(D-1)}{2}(\dot{X})^2 + D\dot{X}\dot{Y} - \frac{2}{D}(\dot{\chi})^2 \right] = -\lambda \exp(B + Y + \frac{D+2}{D}\Phi) \Delta. \quad (3.10)$$

From the  $yy$  component we find

$$\left[ \frac{D(D-1)}{2}(A')^2 + DA'C' - \frac{2}{D}(\varphi')^2 \right] + e^{2(B-C)} e^{2(Y-Z)} \left[ D\ddot{X} + \frac{(D+1)D}{2}(\dot{X})^2 - D\dot{X}\dot{Z} + \frac{2}{D}(\dot{\chi})^2 \right] = 0, \quad (3.11)$$

and finally the off-diagonal  $yr$  equation has the form

$$D(A'\dot{X} - A'\dot{Y} - C'\dot{X}) + \frac{4}{D}\varphi'\dot{\chi} = 0. \quad (3.12)$$

Notice that the jump conditions implied by eqs. (3.8), (3.9) and (3.10) require discontinuous  $\varphi'$ ,  $A'$  and  $C'$ . Apparently, consistent solutions must satisfy

$$\dot{Y} + \frac{D+2}{D}\dot{\chi} = 0, \quad (3.13)$$

so that there is no  $r$  dependence on the right hand side of the jump conditions. Substituting (3.13) into (3.12) gives

$$D^2(A' - C')\dot{X} + ((D+2)DA' + 4\varphi')\dot{\chi} = 0, \quad (3.14)$$

which severely restricts the solutions.

By rescaling the coordinates  $r$  and  $y$  we are free to choose  $B = C$ ,  $Y = Z$ . It turns out that most of the possible solutions of (3.14) are inconsistent with the remaining equations. For instance, taking  $\dot{X} = 0$  which requires  $\dot{\chi} = 0$  or  $4\varphi' = -D(D+2)A'$ , leads to bulk solutions incompatible with the brane matching conditions. Similarly, the choice  $A = C$  and  $4\varphi' = -D(D+2)A'$  is inconsistent. Altogether, we find two consistent solutions, labelled I and II and discussed in the following.

### 3.1. Solution I

We take  $A = C$  and satisfy (3.14) by fixing  $\dot{\chi} = 0$ . Notice that we can set  $Y = 0$ . The eqs. (3.9), (3.10) and (3.11) imply  $\dot{X} = \pm K$ , for some positive constant  $K$ . For concreteness we choose  $\dot{X} = -K$ . The remaining equations simplify to

$$\begin{aligned}\varphi'' + DA'\varphi' &= \frac{D+2}{4}\lambda \exp\left(A + \frac{D+2}{D}\Phi\right) \Delta \\ A'' + D(A')^2 + DK^2 &= -\frac{\lambda}{D} \exp\left(A + \frac{D+2}{D}\Phi\right) \Delta \\ (A')^2 - \frac{4}{D^2(D+1)}(\varphi')^2 + K^2 &= 0.\end{aligned}\tag{3.15}$$

In the bulk these equations are solved by

$$\begin{aligned}A(y) &= \frac{1}{D} \log \left| \sin[DKy + 2\theta] \right| \\ \Phi_{\pm}(y) &= \pm \frac{\sqrt{D+1}}{2} \log \left| \tan\left[\frac{D}{2}Ky + \theta\right] \right| + \Phi_0 \\ X(r) &= -Kr,\end{aligned}\tag{3.16}$$

where in  $A$  we have dropped a constant that can be absorbed in rescaling the coordinates  $x_{\mu}$ . In order to solve the jump conditions we choose

$$\begin{aligned}A(y) &= \frac{1}{D} \log \left| \sin[DKy] \right|, \quad 0 \leq y \leq \frac{L}{2} \\ A(y) &= \frac{1}{D} \log \left| \sin[DK(y-L)] \right|, \quad \frac{L}{2} \leq y \leq L,\end{aligned}\tag{3.17}$$

and similarly for  $\varphi$ . In the interval  $[L, 2L]$  the solution is extended periodically. Matching then requires

$$\cos\left[\frac{D}{2}KL\right] = \mp \frac{2\sqrt{D+1}}{D+2} \quad ; \quad e^{\frac{D+2}{D}\Phi_0} = \frac{K}{\eta(D)\lambda},\tag{3.18}$$

where  $\eta(D)$  is a numerical factor that can be easily evaluated, *e.g.*  $\eta(8) = 0.30$  and  $\eta(4) = 0.51$ . We choose  $\frac{D}{2}KL < \pi/2$ , and correspondingly  $\Phi_-$ , so that the metric has only singularities at  $y = 0, L$ .

By computing the Ricci scalar in string frame one finds divergences at  $y = 0, L$ , so that we have naked singularities in the internal space. However, the dilaton also diverges at these points leading to an infinite string coupling at the singularity. Thus, our next-to-leading order treatment of the string loop expansion breaks down and one might hope that higher loop or non-perturbative effects cure this singularity. After all it is not too surprising that we find these singularities in the solution. Roughly speaking, developing



these singularities is the way gravity can handle a configuration of sources (two positive tensions) that for RR-fields (two positive RR-charges) would be inconsistent.

In the resulting metric with  $D$ -dimensional Poincaré invariance the coordinate  $r$  is also non-compact. In general, we can define an effective size for this coordinate

$$\rho = \int_{-\infty}^{\infty} dr e^{Z_s}, \quad (3.19)$$

where  $Z_s = Z + \frac{2}{D}\chi$  is the corresponding warp factor in the string metric. Since in this solution  $Z = 0$  and  $\dot{\chi} = 0$ , we see that  $\rho$  is unbounded.

Note that in contrast to our situation, in the Randall-Sundrum (RS) set up [18] an exponential warp factor led to localized gravity in one dimension lower. In the RS case this was due to the choice  $X(r) = -K|r|$ , which is not allowed in our case, as it introduces a new singularity in the  $r$  direction.

### 3.2. Solution II

This solution will turn out to be much more interesting and non-trivial than solution I. We only assume  $\dot{X} = \alpha\dot{\chi}$ , with  $\alpha$  constant as required by eq. (3.14). By virtue of variable separation, the bulk equations reduce to

$$\begin{aligned} A'' + D(A')^2 &= -D K^2 \\ \varphi'' + D A' \varphi' &= -D K^2 / \alpha \\ C'' + D A' C' &= -D \mu K^2 \\ \ddot{\chi} + D \dot{X} \dot{\chi} &= D K^2 / \alpha, \end{aligned} \quad (3.20)$$

where  $K$  and  $\mu$  are constants. There are further relations

$$\begin{aligned} 2D A' C' - (D-1) A'' - \frac{4}{D} (\varphi')^2 &= -D K^2 (\mu + 1 + \frac{D+2}{\alpha D}) \\ 2D \dot{X} \dot{\chi} - (D-1) \ddot{X} - \frac{4}{D} (\dot{\chi})^2 &= -D K^2 (\mu - 1 + \frac{D+2}{\alpha D}). \end{aligned} \quad (3.21)$$

After some more computational steps, the solutions to the bulk equations turn out to be

$$\begin{aligned} A(y) &= \frac{1}{D} \log \left| \sin[DKy + 2\theta] \right| \\ C_{\pm}(y) &= \frac{\mu}{D} \log \left| \sin[DKy + 2\theta] \right| \pm \sqrt{\frac{8}{D}} \frac{1}{\alpha D} \log \left| \tan\left[\frac{D}{2}Ky + \theta\right] \right| \\ \Phi_{\pm}(y, r) &= \frac{1}{\alpha D} \log \left| \sin[DKy + 2\theta] \right| \pm \sqrt{\frac{D}{2}} \log \left| \tan\left[\frac{D}{2}Ky + \theta\right] \right| + \\ &\quad \frac{1}{\alpha D} \log \left| \cosh[DKr + \beta] \right| + \Phi_0 \\ X(r) &= \frac{1}{D} \log \left| \cosh[DKr + \beta] \right| \quad ; \quad Y(r) = -\frac{D+2}{\alpha D} X(r). \end{aligned} \quad (3.22)$$

Substituting into eqs. (3.14) and (3.21) determines the constants  $\alpha$  and  $\mu$  to be

$$\mu = \frac{D+1}{2} + \frac{2}{\alpha^2 D^2}, \quad \alpha_{\pm} = \frac{(D+2) \pm \sqrt{(D+8)D}}{D(D-1)}. \quad (3.23)$$

We have absorbed an integration constant in  $C_{\pm}$  in a redefinition of  $K$ . As we will explain, the solution with  $\alpha_-$  leads to diverging lower dimensional quantities, so that in the following we discuss only the case  $\alpha_+$ .

To solve the matching relations we again choose  $A$  of the form (3.17) and similarly for  $C$  and  $\varphi$ . Remarkably, the three jump conditions turn out to be compatible and lead to

$$\cos\left[\frac{D}{2}KL\right] = \mp \sqrt{\frac{8}{D+8}}, \quad e^{\frac{D+2}{D}\Phi_0} = \frac{K}{\kappa(D)\lambda}, \quad (3.24)$$

where the sign corresponds to the free sign in  $C_{\pm}$  and  $\Phi_{\pm}$ . We choose  $\frac{D}{2}KL < \pi/2$  so that the metric, as well as the dilaton, have singularities only at  $y = 0, L$ . Thus, we choose  $\Phi_-$  and  $C_-$  in (3.22). The numerical coefficients  $\kappa(D)$  can be found in Table 1. Consistently, the right hand side of the first equation in (3.24) is smaller than one. The qualitative form of the solutions for  $A, C$  and  $\varphi$  are shown in figures 1-3.

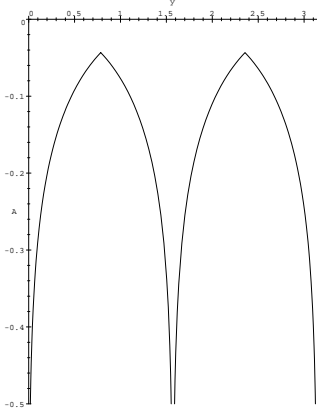


Fig.1:  $A(y)$

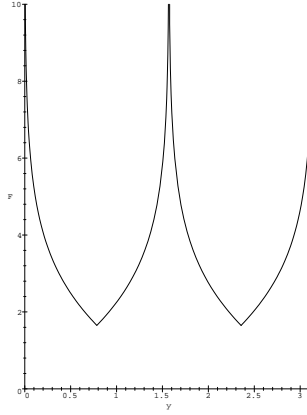


Fig.2:  $\varphi(y)$

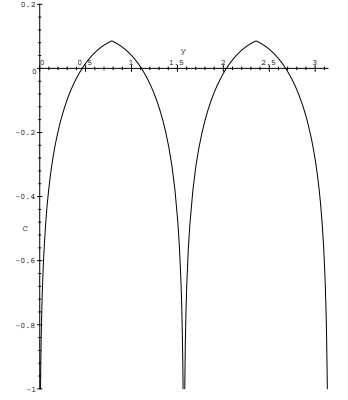


Fig.3:  $C(y)$

In this case the coordinate  $r$  is actually compact. Computing the effective size according to (3.19) gives

$$\rho = \int_{-\infty}^{\infty} dr e^{Z_S} = \epsilon(D) L, \quad (3.25)$$

where the numerical coefficients can be found in Table 1. In contrast to the tree level result, the sizes of the two non-flat directions are correlated.

As in solution I there appear naked singularities and diverging string couplings at  $y \in \{0, L\}$ . The comments made for solution I also apply here. Moreover, we find singularities for  $r \rightarrow \pm\infty$ , where also the string coupling diverges.

#### 4. Effective couplings

Even though we first compactified only the direction  $y$ , the backreaction of the dilaton tadpoles forced us to spontaneously compactify another direction  $r$ . Thus, we do not get an effective theory with  $(D + 1)$  dimensional Poincaré invariance. The best we can hope for is an effective theory with  $D$  dimensional Poincaré symmetry. This of course is very similar to the Randall-Sundrum scenario, where gravity confines to some lower dimension simply by a non-trivial warp factor. Thus, given the solutions found in the previous section we now want to analyze whether gravity and gauge interactions are really confined to the  $D$ -dimensional space-time. To this end we compute the  $D$ -dimensional Planck mass and gauge couplings. After transforming to the Einstein frame these quantities are given by

$$\begin{aligned} M_{Pl}^{D-2} &= M_s^8 V_{8-D} \int_0^{2L} dy \int_{-\infty}^{\infty} dr e^{(D-2)A+B+C+(D-2)X+Y+Z} \\ \frac{1}{g_D^2} &= M_s^5 V_{8-D} \int_0^{2L} dy \int_{-\infty}^{\infty} dr e^{\frac{(D-6)}{D}\Phi} e^{(D-4)A+C+(D-4)X+Z} [\delta(y - \frac{L}{2}) + \delta(y - \frac{3L}{2})]. \end{aligned} \quad (4.1)$$

In the case of solution I, for both quantities the integral in  $r$  diverges as the only  $r$  dependence appears in  $X = -Kr$ . Therefore, the solution I does not lead to a finite effective theory in  $D$ -space-time dimensions.

Contrarily, in the case of solution II,  $M_{Pl}$  turns out to be finite provided we choose  $\alpha_+$ . More concretely, by evaluating the integrals numerically we obtain

$$M_{Pl}^{D-2} = \gamma(D) \frac{M_s^8 V_{8-D}}{K^2}, \quad (4.2)$$

where the numerical coefficients are given in Table 1. On the other hand, for the Yang-Mills coupling we obtain

$$\frac{1}{g_D^2} = \delta(D) \frac{M_s^5 V_{8-D}}{K} e^{\frac{(D-6)}{D}\Phi_0}. \quad (4.3)$$

As can be seen from Table 1, the coefficient  $\delta(D)$  diverges for  $D \in \{8, 9\}$  and is finite only for  $D \leq 6$ .

$D$	$\kappa(D)$	$\epsilon(D)$	$\gamma(D)$	$\delta(D)$
2	1.41	19.73	0.74	0.14
3	1.04	10.98	0.72	0.33
4	0.85	8.07	0.70	0.67
5	0.73	6.60	0.68	1.44
6	0.64	5.72	0.67	5.87
7	0.58	5.12	0.66	$\infty$
8	0.53	4.70	0.65	$\infty$

**Table 1:** *Numerical coefficients.*

Thus, we only get a bona fide effective theory with at most six-dimensional Poincaré symmetry. This is in contrast to supersymmetric vacua, where the number of flat directions is a free parameter. We conclude, that in non-supersymmetric theories the number of flat non-compact directions is not a free parameter, but can be restricted by the dynamics. This hints to an appealing dynamical mechanism to explain why we live in four dimensions.

Finally, let us see whether the solution admits to disentangle the Planck and the string scale. After a further toroidal compactification on  $T^{(D-4)}$  to four flat dimensions, we obtain the following relations for the four dimensional scales

$$M_{Pl}^2 \sim M_s^8 V_{8-D} W_{D-4} L^2, \quad \frac{1}{g_4^2} \sim M_s^{\frac{(4D+16)}{D+2}} V_{8-D} W_{D-4} L^{\frac{8}{D+2}}, \quad (4.4)$$

where  $W_{D-4}$  is the volume of  $T^{(D-4)}$ . Note that these relations differ from the tree level results (2.1). Choosing the gauge coupling of order one implies

$$M_{Pl}^2 \sim M_s^{\frac{4D}{D+2}} L^{\frac{(2D-4)}{D+2}}, \quad (4.5)$$

showing that  $M_s$  is a free parameter as long as we choose the radius  $L$  large enough. We conclude, that large extra dimension scenarios are possible even when the next to leading order quantum corrections to the background are taken into account. Inserting numerical values into (4.5) and choosing  $M_s = 1\text{TeV}$  gives the rough estimates of the internal dimensions shown in Table 2.

$D$	$L$	$(V_{8-D} W_{D-4})^{\frac{1}{4}}$
6	$10^{14}\text{m}$	$10^{-27}\text{m}$
5	$10^{19}\text{m}$	$10^{-30}\text{m}$
4	$10^{30}\text{m}$	$10^{-35}\text{m}$

**Table 2:** *Large extra dimensions.*

Thus, in agreement with the naive tree-level result (2.1), in order to obtain phenomenologically acceptable sizes one has to apply more T-dualities to get D-branes with more transversal directions. However, extrapolating the results presented in [15] and in the present paper, it is a non-trivial question whether the critical dimension for such solutions would be larger than three.

## 5. Conclusions

In this paper we have studied the backreaction of the dilaton tadpole for a prototype model featuring brane supersymmetry breaking. Making a warped ansatz for the metric we have found a highly non-trivial solution to the equations of motion, which allowed to compute finite effective couplings of a lower dimensional theory in flat space provided the dimensions were smaller than the critical value six. We have shown that the naive tree level relations for these couplings in terms of the string scale and the internal geometry change, but that they in principle still allow large extra dimensions and a string scale in the TeV range. However, for  $\overline{D8}$  branes these extra dimensions were unacceptable large, so that one should study models with lower dimensional branes. It would also be interesting to determine the spectrum in this highly curved backgrounds and in particular discuss stability issues.

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